2022 Mathematics Specialist 12: Investigation 2

Take-home part (Main syllabus points 3.3.11-3.3.15)

Consider an object moving such that its position at time t is given by the vector function

$$\boldsymbol{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

We define the following functions related to $\boldsymbol{r}(t)$:

Velocity:

$$\boldsymbol{v}(t) = \begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix}$$

Acceleration:

$$\boldsymbol{a}(t) = \begin{pmatrix} x''(t) \\ y''(t) \end{pmatrix}$$

'Scalar product' of velocity and acceleration:

$$\boldsymbol{v}(t) \cdot \boldsymbol{a}(t) = x'(t)x''(t) + y'(t)y''(t)$$

Investigate the graphs of vector functions of the form:

$$\boldsymbol{r}(t) = \begin{pmatrix} a\cos(bt) \\ c\sin(dt) \end{pmatrix}$$

where a, b, c and d are positive integers. (By the graph we mean the curve defined by the parametric equations $x = a\cos(bt)$ and $y = c\sin(dt)$).

You should focus in particular on the way in which $\boldsymbol{v}(t)$, $\boldsymbol{a}(t)$ and $\boldsymbol{v}(t) \cdot \boldsymbol{a}(t)$ are related to properties of the graph.

Some questions to consider:

- How are the values of a, b, c or d related to when (or whether) $\boldsymbol{v}(t) = \boldsymbol{0}$, $\boldsymbol{a}(t) = \boldsymbol{0}$ or $\boldsymbol{v}(t) \cdot \boldsymbol{a}(t) = 0$?
- If $\boldsymbol{v}(t) = \boldsymbol{0}$, $\boldsymbol{a}(t) = \boldsymbol{0}$ or $\boldsymbol{v}(t) \cdot \boldsymbol{a}(t) = 0$ for some value of t, how is this related to properties of the graph near the point corresponding to that value of t?
- How do other properties of the functions $\boldsymbol{v}(t)$, $\boldsymbol{a}(t)$ and $\boldsymbol{v}(t) \cdot \boldsymbol{a}(t)$ correspond to features of the graph of $\boldsymbol{r}(t)$?

For the take-home part, you are encouraged to use your ClassPad and/or graphing software to investigate the graphs of these functions.

In the validation test you will be allowed two A4 pages with notes on both sides, and a scientific calculator, but NO ClassPad. A formula sheet will be provided.